

Sieve of Eratosthenes: An Ancient Algorithm to Discover Prime Numbers

Reporting Category Number and Number Sense

Topic Identifying and describing prime and composite numbers

Materials

- Centimeter grid paper
- Rectangles Chart (attached)
- Hundred Chart (attached)
- Color pencils

Vocabulary

rectangle, square, factor, prime number, composite number, dimension

Student/Teacher Actions (what students and teachers should be doing to facilitate learning)

1. Explain to students that they will be investigating characteristics of certain types of numbers by drawing rectangles on grid paper. Model outlining a 1×2 inch rectangle on a sheet of grid paper. Explain that this rectangle has been created with two square units, and has dimensions of 1 linear unit and 2 linear units. (Depending on the orientation of the rectangle, the dimensions could also be 2 and 1.) On the Rectangles Chart, next to the number 2, record the dimensions (1, 2) in the factors column. Explain to students that the whole number dimensions of this rectangle are also called factors of the number.
2. Have students create a rectangle with three square units on the grid paper and record the dimensions (factors) on the Rectangles Chart next to the 3. Point out to students that, so far, they have only been able to create one rectangle for each number and that the dimensions have been the number and 1. Practice with one more rectangle: Have students create the possible rectangles with four square units and share (one with dimensions of 4, 1; one with dimensions of 2, 2).
3. Have groups of students work cooperatively to complete the Rectangles Chart (attached) by continuing to create rectangles and record the dimensions and the number of rectangles that can be created for square units for the numbers 5 through 25. Each group can be assigned to work with a few of these numbers, and the data can be collected from each group once the task is completed.
4. Record all the data on the overhead chart, and have students discuss what they notice about the different numbers. They should notice that only one rectangle could be created with some of the numbers. Have students locate these numbers on the Rectangles Chart and discuss the characteristics of the factors of these numbers. Note that the factors of

these numbers (dimensions of these rectangles) are only the number and 1. Ask students if they know the name of these types of numbers (prime). *Prime numbers* are defined as having exactly two unique factors. Explain that the other numbers, those that have more than two factors, are called *composite*.

5. Share with students that the number 1 has one factor (itself) and forms one rectangle (a 1×1 square); it is classified by mathematicians as a special number and is neither prime nor composite.
6. Have the students look at the rectangles that were made to represent 1, 4, 9, 16, and 25. Point out that one of the rectangles formed with the factors of these numbers is also a square.
7. For the second part of this lesson, tell students that Eratosthenes was an ancient Greek mathematician who studied prime and composite numbers. He used a method now called the Sieve of Eratosthenes. Distribute the Hundred Chart to the students, and have them proceed through the following directions to locate the prime and composite numbers.
 - Since the number 1 is not prime, color it purple on the Hundred Chart (attached).
 - Circle the first prime number, 2, with your pencil. Color every multiple of 2 on your chart yellow. Do not color the number 2.
 - Circle the next prime, 3. Color the multiples of 3 on your chart red. (Some multiples of 3, such as 6 and 12, may already be colored yellow. Ignore these and look for the uncolored multiples. When you finish, you should have 16 red squares.)
 - Circle the next prime, 5. Color any uncolored multiples of 5 blue. (You should have 6 blue squares.)
 - Circle the prime number, 7. Color any uncolored multiples of 7 green. (You should have only 3 green squares.)
 - Count the uncolored squares on your chart. Can you find 25 of them? If you can, then you have sifted out all the prime numbers under 100.

Assessment

- **Questions**
 - What is the difference between a prime number and a composite number? Give examples of both, and draw pictures to help your explanation.
 - How can you determine whether a number is prime?
- **Journal/Writing Prompts**
 - Record in your math journal the characteristics of prime and composite numbers, and describe using the Sieve of Eratosthenes to find the primes.
 - Explain what makes prime numbers special. Choose your favorite prime number and explain why it is your favorite.
- **Other**
 - Give students a blank Hundred Chart, and have them identify all the prime numbers. They may choose to replicate the Sieve of Eratosthenes process.
 - Give students a list of random numbers, such as 42, 27, 31, 56, and 83. Ask them to tell you if each is prime or composite and to explain why.

Extensions and Connections (for all students)

- Extend exploration of prime and composite numbers to numbers greater than 100. Give students a chart of numbers from 101 to 200, and see if they can identify the primes. Ask students to justify how they know those numbers are prime.
- Challenge students to explain how to determine whether a number is divisible by 2, 3, 4, 5, 6, 8, and 9. Ask them to develop divisibility rules for each of these divisors.

Strategies for Differentiation

- Use color tiles to assist students in creating rectangles for numbers. For example, students would need 16 tiles to explore and create rectangles that can be made for the number 16.
- Model creating rectangles and squares for numbers, and model completing the charts in this lesson.

Rectangles Chart

Name _____ Date _____

Number	Factors	Number of Rectangles	Prime or Composite
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			
21			
22			
23			
24			
25			

Hundred Chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100